Application of the Nonuniform FDTD Technique to Analysis of Coaxial Discontinuity Structures

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***Abstract*—In this paper, we employ the nonuniform finite-difference time-domain (NUFDTD) technique to accurately model discontinues in complex coaxial configurations. We take advantage of the azimuthal symmetry of the structure to reduce the original problem into an equivalent two-dimensional one, and we do this by projecting the three-dimensional Yee cell onto two-dimensional planes. Numerical results are presented for various discontinuities to illustrate the application of the method. We show that the NUFDTD technique yields results that are in excellent agreement with the mode-matching method.**

***Index Terms*—Coaxial discontinuity structure, nonuniform FDTD.**

I. Introduction

The finite-difference time-domain (FDTD) method is a useful computational technique [1]–[5] for carrying out a broad variety of electromagnetic simulations over a broad frequency range. In this paper, we focus on coaxial discontinuity type of problems that are frequently encountered in many microwave applications. Such problems are often attacked *via* the finite-element method (FEM) or, in special cases, by using quasi-analytic approaches [6]–[9]. The conventional FDTD method on a uniform mesh, however, is not very well suited for analyzing the geometries of interest in this paper, i.e., because the geometrical dimensions of the structure are not always commensurate with the discertization employed, unless an extremely fine mesh is used to model it accurately, at a prohibitively large cost of computational memory and time.

One drawback of the conventional nonuniform finite-difference time-domain (NUFDTD) scheme, however, is that it is difficult to deal with the interfaces, edges, and corners involving dissimilar media. To circumvent this difficulty, we introduce an alternative version of the FDTD scheme, in which we define the parameters of the medium at the center of the FDTD cell, as opposed to the locations of the E- and H-fields.

For the coaxial discontinuity structures considered in this paper, the azimuthal symmetry enables us to employ a two-dimensional lattice, which is derived by projecting the three-dimensional (3-D) Yee cell in the original cylindrical coordinates onto a two-dimensional plane. This not only renders the algorithm numerically efficient, but also reduces the demands on the CPU memory considerably. Yet another tactic we employ to reduce the computational time is to use an algorithm that requires only a single run of the FDTD to compute the S-parameters, rather than two passes normally employed in the conventional FDTD approach.

To validate the approach, we compute the reflection coefficients of several coaxial discontinuities and compare them with those derived by using the mode-matching method.

II. FDTD Method for Coaxial Discontinuity Structures

A. FDTD Formulation for Circularly Symmetric Structures

For circularly symmetric structures, we can assume that the electromagnetic fields have either a $\sin(m\phi)$ or $\cos(m\phi)$ type of angular variation. Hence, this behavior can be factored out from the field equations, and the original 3-D problem can be reduced to an equivalent two-dimensional one. As mentioned in the previous section, we define the parameters of medium at the center of the FDTD cell instead of at the location of the corresponding fields. We introduce an alternative version of the nonuniform FDTD update algorithm to simplify the numerical computation. The two representative update equations, e.g., E_r and H_z, are written as

$$\varepsilon_1 = \Big(\Delta z(j)\cdot\varepsilon_r(i,\,j) + \Delta z(j-1)\cdot\varepsilon_r(i,\,j-1)\Big)\Big/ \Big(\Delta z\,(j) + \Delta z\,(j-1)\Big)$$

$$\sigma_1 = \Big(\Delta z\,(j)\cdot\sigma_r\,(i,\,j) + \Delta z\,(j-1)\cdot\sigma_r\,(i,\,j-1)\Big)\Big/ \Big(\Delta z\,(j) + \Delta z\,(j-1)\Big)$$

$$\begin{aligned}E_r^{n+1}\,(i,\,j) &= \frac{\left(1-\dfrac{\sigma_1\cdot\Delta t}{2\varepsilon_1}\right)}{\left(1+\dfrac{\sigma_1\cdot\Delta t}{2\varepsilon_1}\right)}E_r^n\,(i,\,j) - \frac{\dfrac{\Delta t}{\varepsilon_1}}{\left(1+\dfrac{\sigma_1\cdot\Delta t}{2\varepsilon_1}\right)}\\ &\quad\cdot\left[\frac{H_\phi^{n+1/2}\,(i,\,j)-H_\phi^{n-1/2}\,(i,\,j-1)}{0.5\,(\Delta z\,(i,\,j-1)+\Delta z\,(i,\,j))}\right] - \frac{\dfrac{m\Delta t}{\varepsilon_1}}{\left(1+\dfrac{\sigma_1\cdot\Delta t}{2\varepsilon_1}\right)}\\ &\quad\cdot\frac{H_z^{n+1/2}\,(i,\,j)}{r_{i+1/2}}\end{aligned} \tag{1}$$

$$\mu_3 = \Big(\Delta z\,(j)\cdot\mu_z\,(i,\,j)+\Delta z\,(j+1)\cdot\mu_z\,(i,\,j+1)\Big)\Big/ \Big(\Delta z\,(j)+\Delta z\,(j+1)\Big)$$

$$\begin{aligned}H_z^{n+1/2}\,(i,\,j) &= H_z^{n-1/2}\,(i,\,j) - \frac{m\Delta t}{\mu_3\cdot r_i}E_r^n\,(i,\,j) + \frac{\Delta t}{\mu_3}\\ &\quad\cdot\left[\frac{E_\phi^n\,(i+1,\,j)-E_\phi^n\,(i,\,j)}{0.125(\Delta r(i-1,\,j)+6\Delta r(i,\,j)+\Delta r(i+1,\,j))\cdot r_{i+1/2}}\right].\end{aligned} \tag{2}$$

where $r_i = (i-(1/2))\Delta r$. The advantage of using the alternative FDTD algorithm in preference to the conventional one is that we calculate the effective parameters of the medium before updating the fields, and this leads us to an universal formula for the interfaces, edges, and corners, with little additional computational burden required to compute the effective parameters at the interfaces, etc.

B. S-Parameter Calculation

The conventional approach to computing the scattering parameters using the FDTD requires that the FDTD algorithm be run twice—the first time to set the reference, *viz.* the incident field, and the second time to find the total field. In this paper, we employ a more efficient technique [10], described below, which only requires a single pass of the FDTD. We begin by expressing the time-domain voltage, sampled

Manuscript received August 3, 1999.

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Publisher Item Identifier S 0018-9480(01)00012-6.

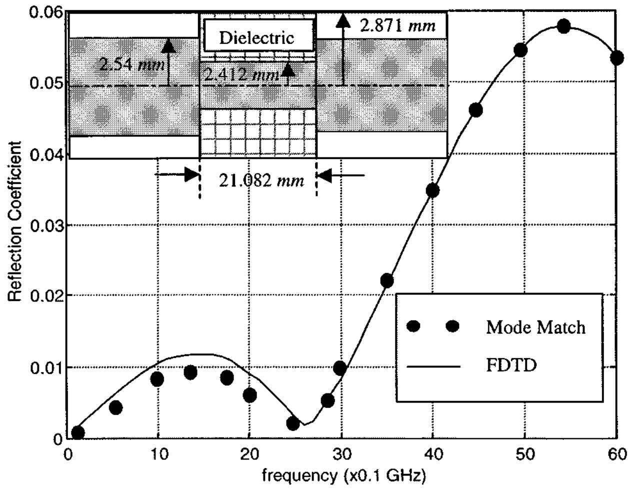

Fig. 1. Comparison of the FDTD and mode-matching results for problem 1.

at the source end, as

$$V_{zi}(n\Delta t) = \sum_{j=0}^{r_{\max}} E_{r_j}\Big(z_i,\, r_j,\, n\Delta t\Big)\Delta r(j), \qquad i = 1,\, 2,\, 3,\, 4 \tag{3}$$

where z_i is the sampled voltage position and E is the computed electric field. The four measurement sampled are taken at locations that are two cells apart, at 10, 12, 14, and 16 cells away to the left-hand side of the first discontinuity. Next, we take the fast Fourier transform (FFT) of the time-domain voltage samples to obtain the $\tilde{V}_{zi}(n\Delta f)$'s in the frequency domain. We then express these voltages as summations of transmitted and reflected voltages in the form

$$\tilde{V}_{zi}(n\Delta f) = Ae^{-jBz_i} + Ce^{jDz_i}, \qquad i = 1,\, 2,\, 3,\, 4. \tag{4}$$

Finally, by using the Prony's method, we calculate the amplitude of the reflected and transmitted waves to compute the desired S-parameter.

Before concluding this section, we point out that the fields vary rapidly in the neighborhood of the discontinuities, and it becomes necessary to use a fine mesh in these regions in order to calculate the S-parameters accurately.

III. Numerical Results

In this section, we apply the improved nonuniform-mesh FDTD to calculate the scattering parameters of coaxial discontinuity structures. In all of the examples presented in this paper, the r- and z-directions are nonuniformly divided into 70 and 260 cells, respectively. The outer and inner surfaces are perfect conductors. The two faces in the z-direction are truncated by a second-order Mur's absorbing boundary condition [11]. The source and observation points are located at the left-hand side of discontinuity structure. The first example considered is depicted in the insert in Fig. 1. The permittivity of the loaded dielectric is $2.038\varepsilon_0$. The maximum and minimum cell sizes in the r-direction are 1.23 and 0.96 mm, respectively, while they are 1.38 and 0.125 mm in the z-direction. The ratio of the adjacent cells of the nonuniform grid is 1.08. A fine mesh is used in the regions that include the interface between the two different materials. The reflection characteristic of the discontinuity structure is plotted in Fig. 1, where the FDTD results are also compared with those obtained via the mode-matching method. Good agreement between the two is noted.

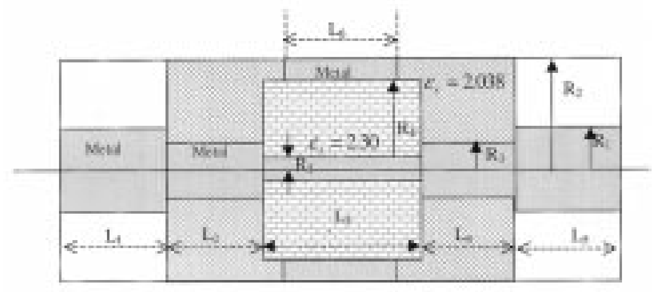

Fig. 2. Geometry of the coaxial discontinuity problem (problem 2).

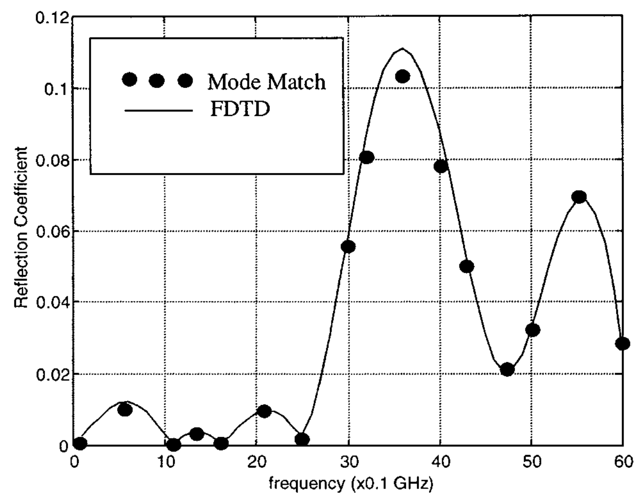

Fig. 3. Comparison of FDTD and mode-matching results for Case 1.

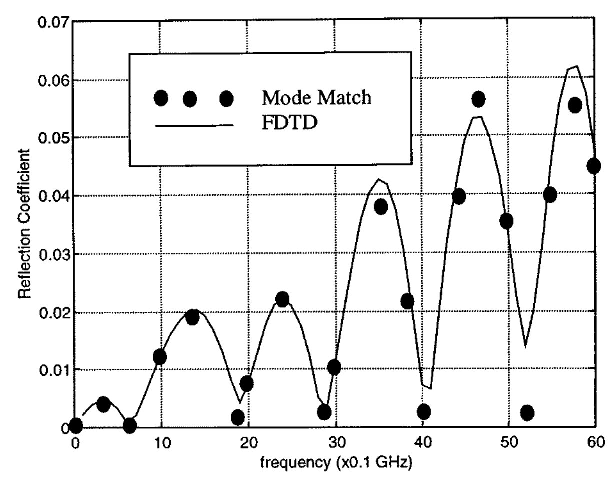

Fig. 4. Comparison of the FDTD and mode-matching results for Case 2.

For the next example, we investigate the effect of dielectric loading on the reflection coefficient of a coaxial discontinuity structure, shown in Fig. 2. For all of the cases, the radii R_1, R_2, R_3, R_4, and R_5 are 3.10, 7.14, 2.18, 5.10, and 1.50 mm, respectively. For Case 1, the lengths L_1, L_2, L_3, L_4, L_5, and L_6, are 25.40, 31.75, 25.40, 31.75, 25.40, and 25.40 mm, respectively, and the reflection coefficient of the loaded structure is plotted in Fig. 3. Once again, an excellent agreement between the FDTD and mode-matching methods is observed for this case,

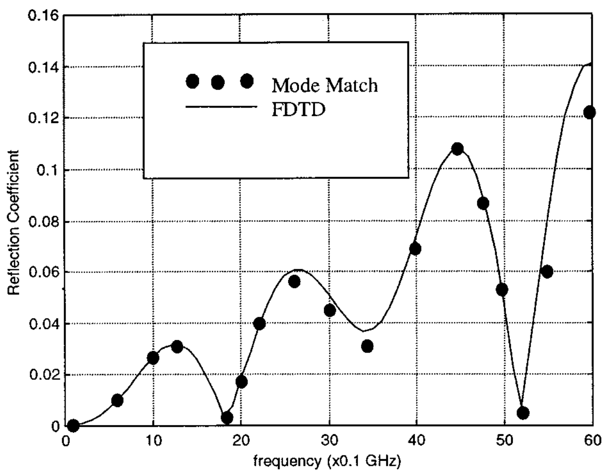

Fig. 5. Comparison of the FDTD and mode-matching results for Case 3.

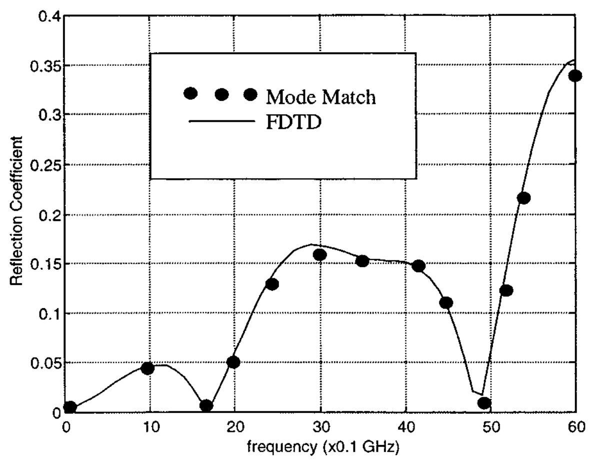

Fig. 6. Comparison of FDTD and mode-matching results for Case 4.

as well as for the three other cases presented below with different parameters. For Case 2, the corresponding lengths are 25.40, 30.34, 28.21, 30.34, 25.40, and 25.4 mm, respectively, and the plots are presented in Fig. 4. Next, for Case 3, the lengths are 25.40, 28.94, 31.02, 28.94, 25.40, and 25.40 mm and the results are plotted in Fig. 5. Finally, Fig. 6 displays the results for the case where the lengths are 25.40, 26.12, 36.65, 26.12, 25.40, and 25.40 mm. We note that the choice of parameters in Case 1 provides a good match in the range of 0–2.5 GHz. However, to obtain a good performance over a wider band of 0–6 GHz, the parameters of Case 2 should be used.

Before closing, it is worthwhile to mention that it takes only 7 min 20 s on an Intel PII 266 computer to run a total 4096 time steps that are adequate for extracting accurate results for the problems investigated in this paper. This is far less time than that required in the full 3-D FDTD formulation, which does not take advantage of the azimuthal symmetry of the structure.

IV. Conclusions

In this paper, we have presented a NUFDTD algorithm for computing the reflection coefficients of coaxial discontinuities possessing azimuthal symmetry. The technique is numerically efficient because it can deal with the interfaces, edges, and corners in an efficient manner, and because it only requires a single FDTD run as opposed to two needed in the conventional approaches. We have shown that results are in good agreement with the mode-matching method.

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